بسم الله الرحمن الرحيم

سبحانك لا علم لنا إلا ما علمتنا إنك أنت العليم الحكيم

Physics B (B 1032) Waves, Light and thermodynamics

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كلية الهندسة ببنهـــــا
قسم العلوم الهندسية الأساسية

PHYSICS B (B1032)

Lecture 1

Wave Motion

By: Prof Dr Tarek Abdolkader

OUTLINE

- **1.1 What is a wave?**
- **1.2 Types of Wave Motion**
- **1.3 Mathematical description of a wave**
- **1.4 Harmonic Waves and their properties**
- **1.5 Wave in a stretched string**

1.1 What is a Wave?

- **Wave is the transfer of a change in a physical quantity from point to point with time**

- **الموجة تنشأ من انتقال التغير في كمية فيزيائية من مكان إلى آخر مع الزمن**

- **In a wave, energy and information are transferred through space without the transfer of matter**

- **الموجة تنقل المعلومات و الطاقة بدون انتقال المادة**

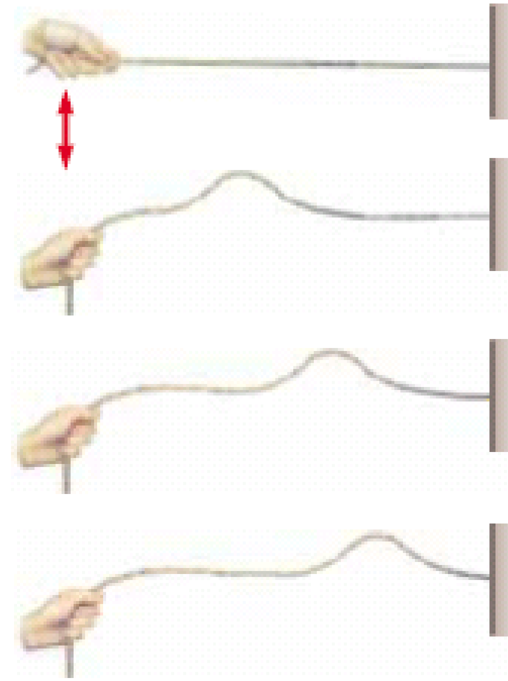
Wave Motion

1.2 Types of Waves

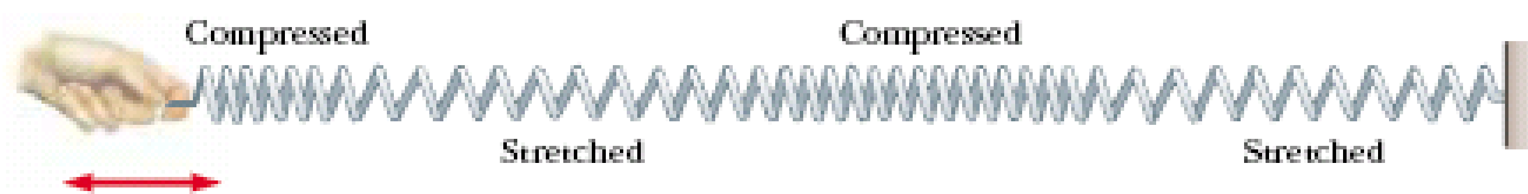
According to the need for medium:

- **Mechanical waves need a medium**

  Examples:
  - Wave in a stretched string
  - Wave in a stretched spring
  - sound wave
  - water wave



Wave in a stretched string



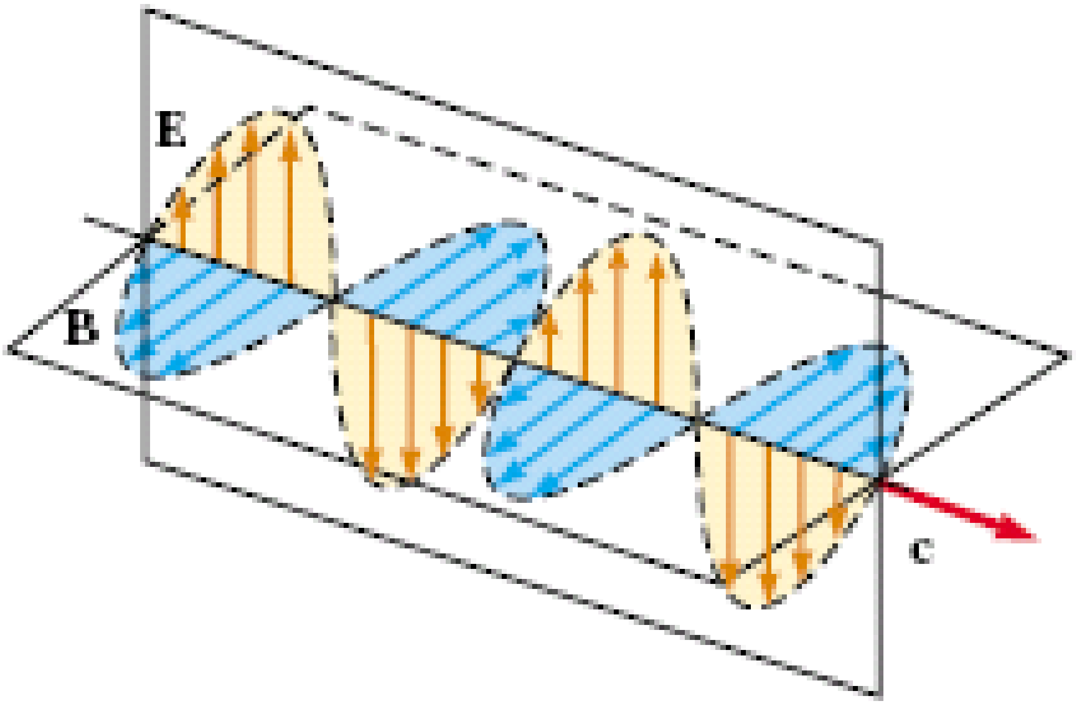
Wave in a stretched spring

1.2 Types of Waves

According to the need for medium:

- **Electromagnetic waves do not need a medium**

  Examples: light, x-ray



Electromagnetic Wave

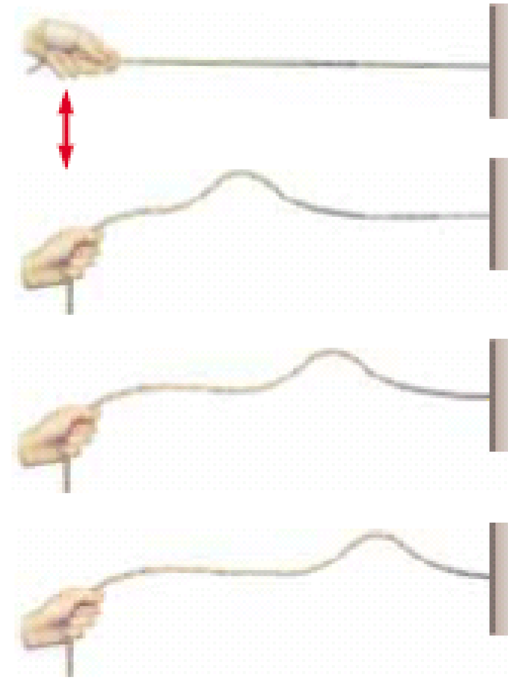
1.2 Types of Waves

According to the direction of propagation:

- **Transverse wave:**

  The physical change is perpendicular to the direction of propagation

  Example: Wave in a stretched string



- **Longitudinal wave:**

  The physical change is parallel to the direction of propagation

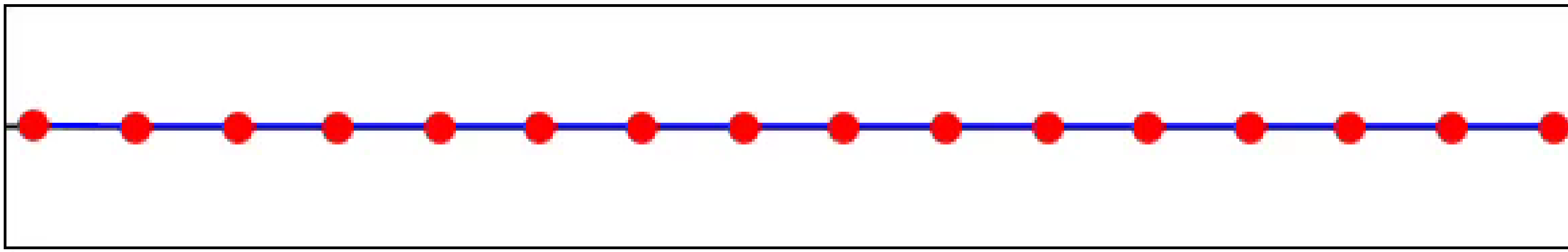
  Example: Wave in a stretched spring, Sound Wave

1.2 Types of Waves

According to the direction of propagation:

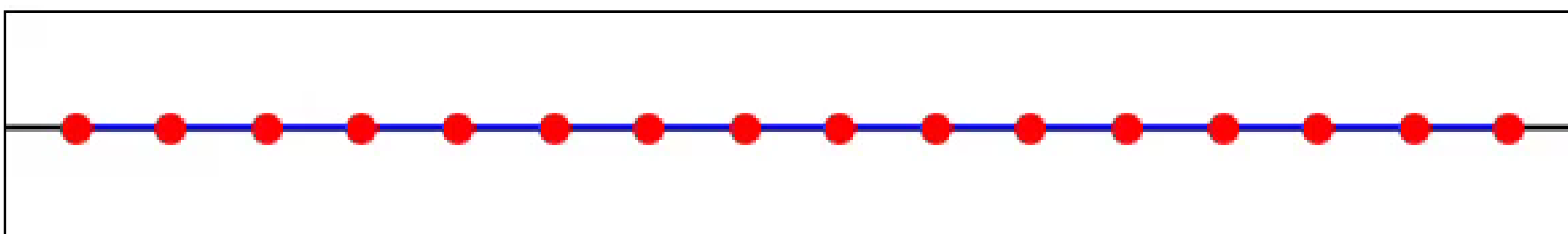
- **Transverse wave:**

  The physical change is perpendicular to the direction of propagation



- **Longitudinal wave:**

  The physical change is parallel to the direction of propagation



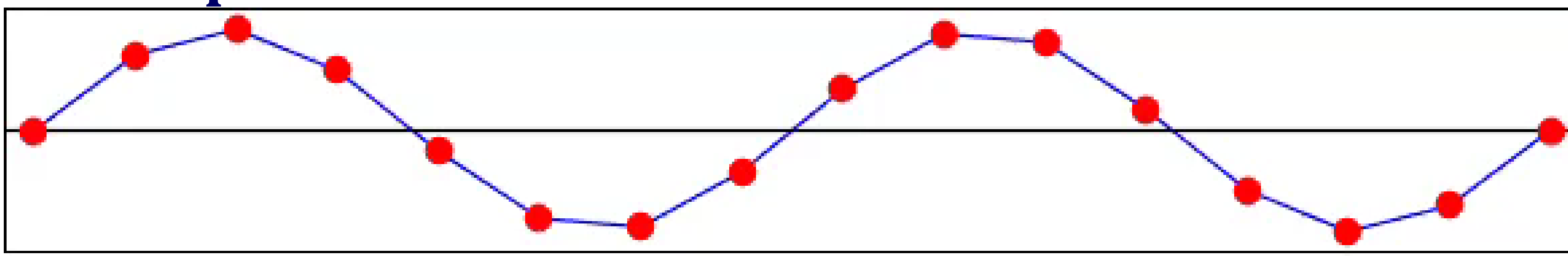
1.2 Types of Waves

According to the nature of change:

- **Periodic waves:**

  the change is continuous and periodic in nature

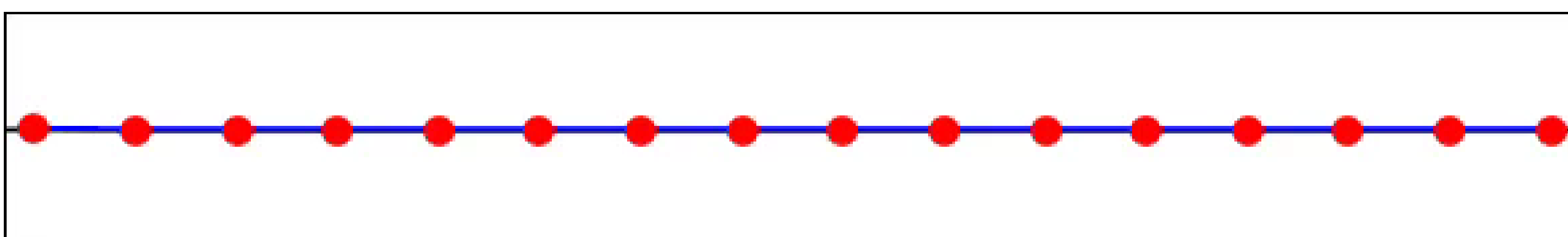
  Examples: sinusoidal waves



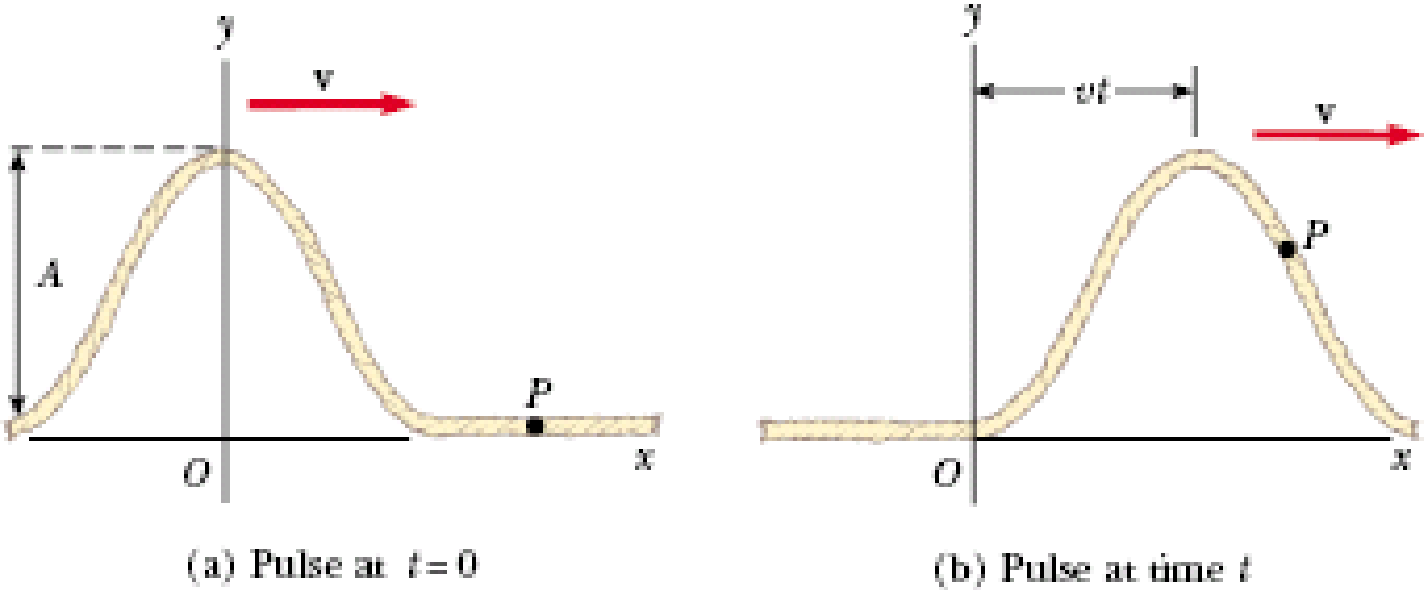
- **Aperiodic waves**

  the change is not periodic

  Examples: disturbance propagation



1.3 Mathematical Description of a Wave



$$y(x,0) = f(x) \quad \& \quad y(x,t) = y(x-vt,0)$$

$$\rightarrow \quad y(x,t) = f(x-vt)$$

1.3 Mathematical Description of a Wave

For a wave propagating in the positive x direction

$$y(x,t) = f(x - vt)$$

For a wave propagating in the negative x direction

$$y(x,t) = f(x + vt)$$

$f(x)$ is called the wave form

$y(x,t)$ is called the wave function

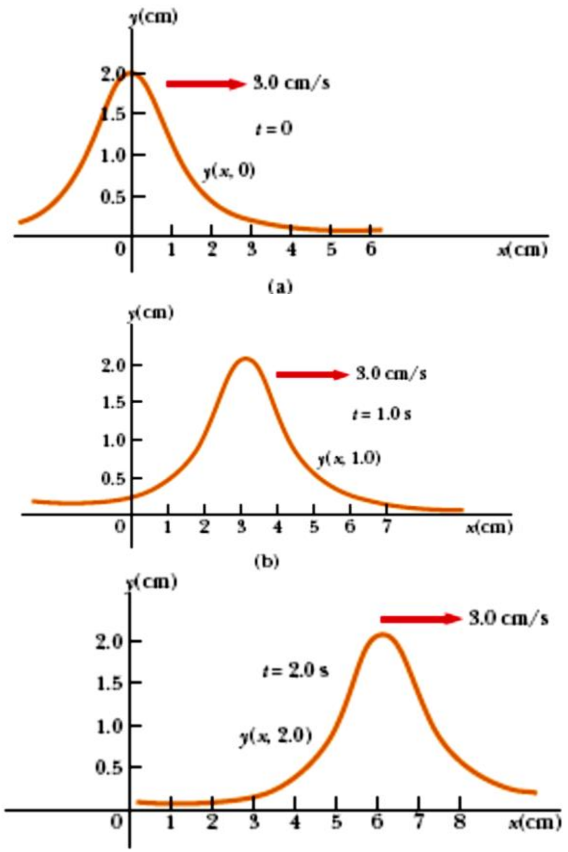
1.3 Mathematical Description of a Wave

EXAMPLE ***A Pulse Moving to the Right.*** See Example 1.1 in the textbook

A pulse moving to the right along the *x* axis is represented by the wave function:

$$y(x,t) = \frac{2}{(x-3t)^2+1}$$

(i) Find the wave form of this wave, (ii) the wave velocity, and (iii) Plot the wave function at $t = 0$, $t = 1.0$ s, and $t = 2.0$ s.



SOLUTION

(i) The equation is in the form $y(x, t) = f(x - vt)$ where

$$f(x) = y(x,0) = \frac{2}{x^2+1}$$

(ii) From the equation of the wave, $v = 3$ cm/s

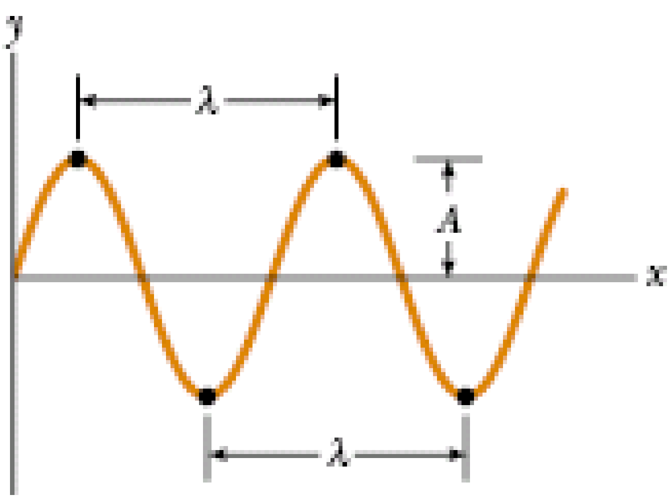
(iii) $$y(x,0) = \frac{2}{x^2+1} \qquad y(x,1) = \frac{2}{(x-3)^2+1} \qquad y(x,2) = \frac{2}{(x-6)^2+1}$$

1.4 Harmonic Waves and their properties

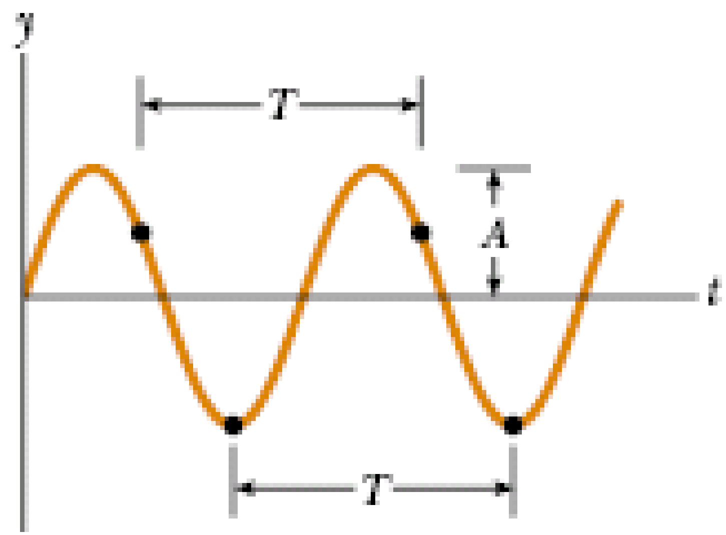
The equation of a wave is generally $y(x,t) = f(x \mp vt)$

For a sinusoidal waveform $f(x) = A\sin(kx + \phi)$

The wave is a called harmonic wave and the wave function is given as: $y(x,t) = A\sin[k(x \mp vt) + \phi]$



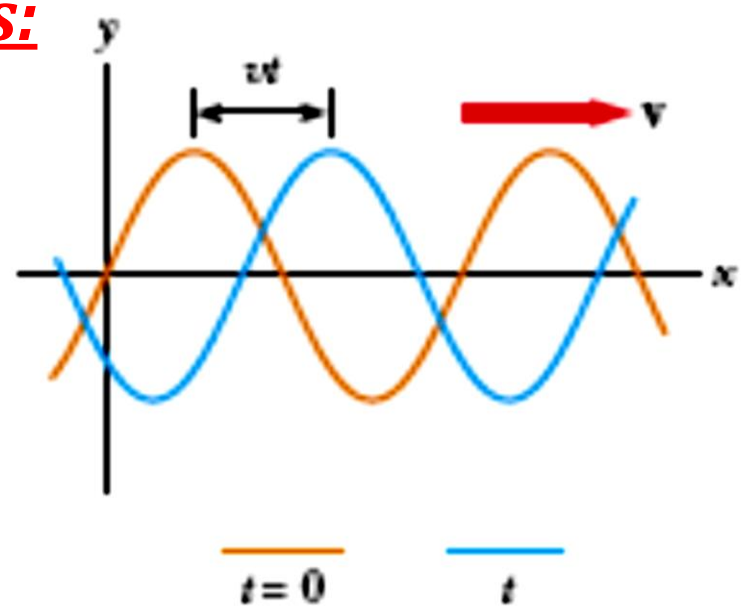
A snapshot at $t = 0$



The variation with time at $x = 0$

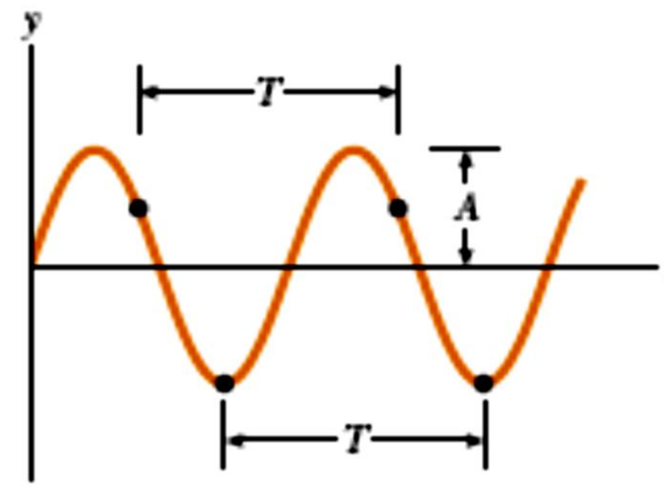
Properties of harmonic waves:

$$y(x,t) = A\sin[k(x \mp vt) + \phi]$$

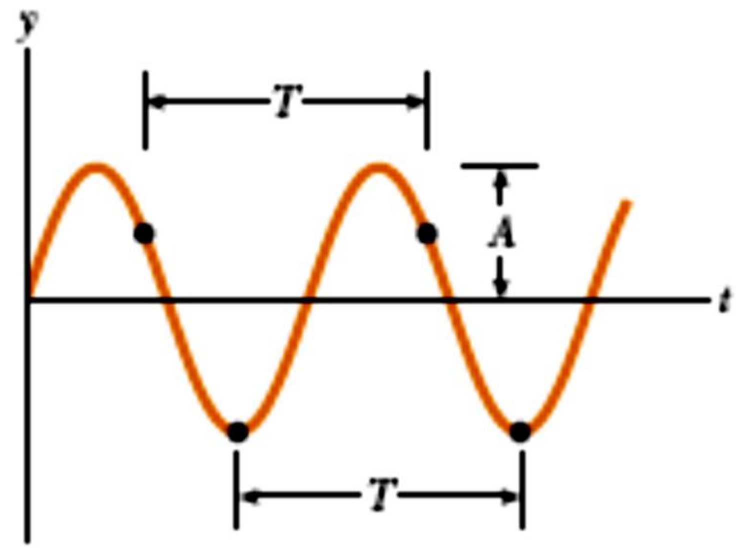


- The ***amplitude*** **(*A*)** of the wave is the maximum value of the wave at any time

- The ***periodic time*** **(*T*)** is the time required to complete one period of the wave. It is the time between any two identical points of the wave (two points of the same phase).



1.4 Harmonic Waves and their properties



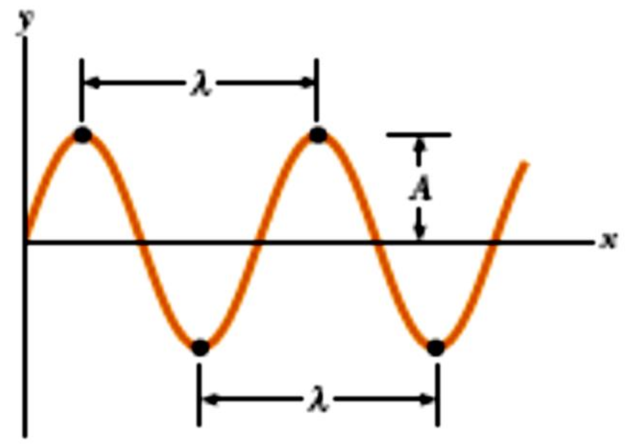
- The ***frequency*** **(*f*)** of the harmonic wave has either of two definitions:

1. It is number of periods made in time unit.
2. It is the number of crests or troughs, or any other point of a certain phase on the wave that pass a given point in a unit time.

The frequency of a sinusoidal wave is related to the period by the expression,

$$f = \frac{1}{T}$$

1.4 Harmonic Waves and their properties



- The ***wavelength* (λ)** of the harmonic wave has either of two definitions:
  1. It is value of the displacement of the wave through one periodic time.
  2. It is the minimum distance between two points of the same phase.

- For one period of the wave the displacement of the wave is λ and the time taken is T, thus,

The ***velocity (v)*** of the wave (or the wave speed) is

$$v = \text{displacement} / \text{time} = \lambda / T = \lambda\ f$$

1.4 Harmonic Waves and their properties

$$f(x) = A\sin(kx + \phi)$$

- From the definition of λ, the waveform completes one period in a distance $x = \lambda$. A complete period corresponds to an angle 2π,

$$k\lambda = 2\pi \quad \rightarrow \quad k = 2\pi / \lambda$$

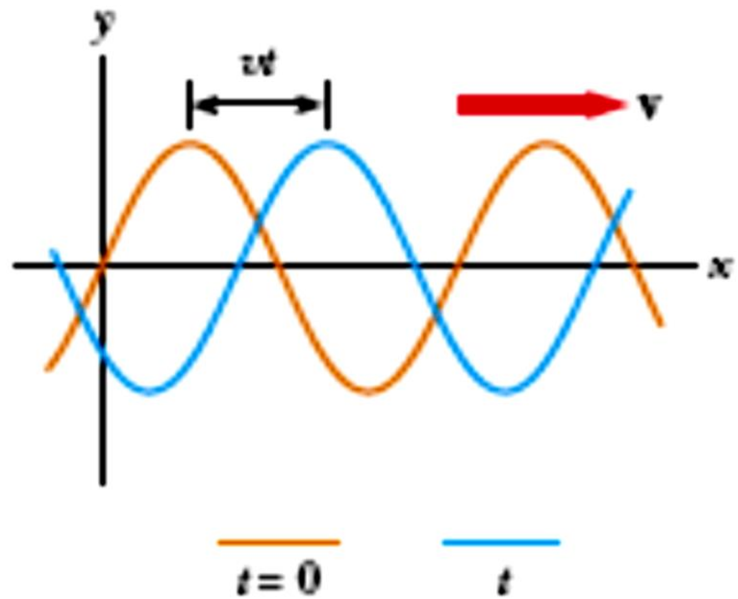
k is called the ***angular wave number*** and has S.I. units of rad /m or m^{-1}.

- The ***angular frequency (ω)*** is defined as the phase angle swept in unit time. Since in a periodic time *T*, a complete period with angle 2π is swept, thus, $\omega = 2\pi / T \quad \rightarrow \quad \omega = 2\pi f$

we can write also, $v = \omega / k$

1.4 Harmonic Waves and their properties

$$f(x) = A\sin(kx + \phi)$$



For a wave propagating in the positive *x*-direction,

$$y(x) = A\sin\left[k(x - vt) + \phi\right]$$

But $k\,v = \omega$ ➔ $$y(x) = A\sin(kx - \omega t + \varphi)$$

Or generally, $$y(x) = A\sin(kx \mp \omega t + \varphi)$$

- The negative sign is for a wave propagating in the positive *x*-direction,
- The positive sign is for a wave propagating in the negative *x*-direction.

Summary:

A harmonic wave $$y(x,t) = A\sin(kx \mp \omega t)$$

has the following parameters:

- **The amplitude (A)** **(max displacement)**
- **The angular wave number (k) = $2\pi / \lambda$**
- **The angular frequency (ω) = $2\pi f = 2\pi / T$**

  (The angle traversed in one period)
- **The wave speed (v) = $\lambda f = \omega / k$**

1.4 Harmonic Waves and their properties

EXAMPLE

A harmonic wave represented by the equation:

$$y(x,t) = 1.75\sin(0.5\pi x - 500\pi t)$$

Where all distances in m and time is in seconds.

1- Plot the wave function at $t = 0$ and at $t = 0.001$ s

2- Find the wavelength, frequency, periodic time, and the velocity of the wave.

SOLUTION

EXAMPLE ***A Traveling Sinusoidal Wave*** See Example 1.2 in the textbook

A sinusoidal wave traveling in the positive x direction has an amplitude of 15.0 cm, a wavelength of 40.0 cm, and a frequency of 8.00 Hz. The vertical position of an element of the medium at $t = 0$ and $x = 0$ is also 15.0 cm, as shown in Figure.

a) Find the wave number , period , angular frequency ,and speed of the wave.
b) Determine the phase constant and write an expression for the wave function.

SOLUTION

1.4 Harmonic Waves and their properties

Important Notes:

1. The wave velocity or the wave speed (v) depends only on the properties of the medium in which the wave propagates and does not depend on the frequency of the wave.

2. The wave velocity is not the velocity of the particles of the medium but it is the velocity of any feature of the waveform.

3. The constant phase depends on the initial conditions of the wave (the starting time and the starting distance).

1.4 Harmonic Waves and their properties

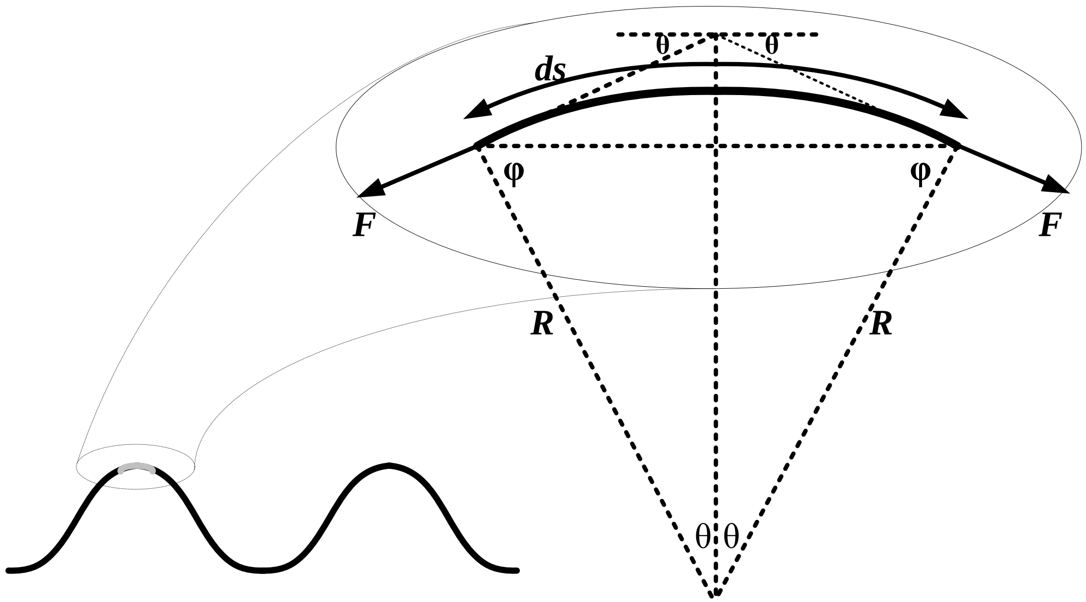
Quick Quiz 1.1 A sinusoidal wave of frequency f is traveling along a stretched string. The string is brought to rest, and a second traveling wave of frequency $2f$ is established on the string. If the wavelength and the wave speed in the first case are λ and v, respectively, then,

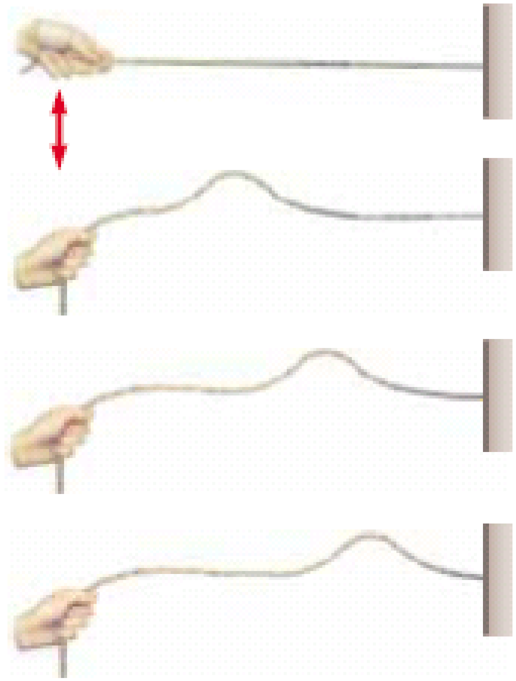
- The wavelength in the second case is ……..
- The wave velocity in the second case is ……..
- The wave number in the second case is ……..
- The angular frequency in the second case is ……..

Quick Quiz 1.2 Find the wave velocity, wavelength, wave number, angular frequency of the wave given by:

$$y(x,t)=\frac{2}{(x-3t)^2+1}$$

1.5 Wave in a stretched string



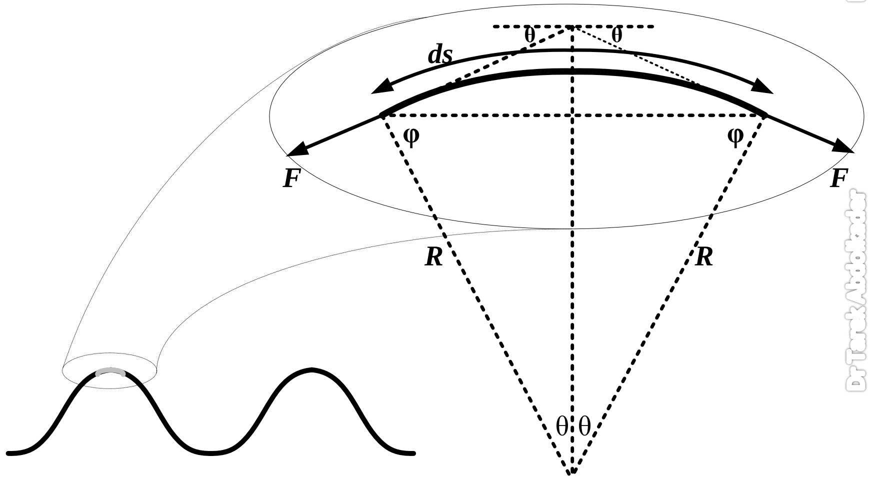


The force in the radial direction = $2\,F \sin\theta \approx 2\,F\,\theta$

The centripetal acceleration = v^2 / R

1.5 Wave in a stretched string

Applying Newton 2^{nd} law:



$$F = m\,a$$

$$2\,F\,\theta = (\Delta m)\,v^2 / R$$

$$\Delta m = \mu\,\Delta\,s = \mu\,R\,(2\,\theta)$$

$$v = \sqrt{F/\mu}$$

1.5 Wave in a stretched string

Quick Quiz Suppose you create a pulse by moving the free end of a taut string up and down once with your hand beginning at $t = 0$. The string is attached at its other end to a distant wall. The pulse reaches the wall at time t. Which of the following actions, taken by itself **decreases the time interval that it takes for the pulse to reach the wall?**

(More than one choice may be correct)

(a) moving your hand more quickly, but still only up and down once by the same amount
(b) moving your hand more slowly, but still only up and down once by the same amount
(c) moving your hand a greater distance up and down in the same amount of time
(d) moving your hand a lesser distance up and down in the same amount of time
(e) using a heavier string of the same length and under the same tension
(f) using a lighter string of the same length and under the same tension
(g) using a string of the same linear mass density but under decreased tension
(h) using a string of the same linear mass density but under increased tension.

EXAMPLE ***The Speed of a Pulse on a Cord*** See Example 1.3 in the textbook

A uniform cord has a mass of 0.300 kg and a length of 6.00 m. The cord passes over a pulley and supports a 2.00-kg object. Find the speed of a pulse traveling along this cord.

SOLUTION

1.5.2 Transverse Velocity:

The transverse velocity v_y is the velocity of the vertical displacement of the string

$$y(x) = A\sin(kx - \omega t)$$

$$v_y = \left.\frac{dy}{dt}\right|_{x=constant} = \frac{\partial y}{\partial t} = -A\omega\cos(kx - \omega t)$$

The transverse acceleration a_y is the vertical acceleration of the string

$$a_y = \left.\frac{dv_y}{dt}\right|_{x=constant} = \frac{\partial v_y}{\partial t} = -A\omega^2\sin(kx - \omega t)$$

Note that the maximum transverse velocity and acceleration are:

$$(v_y)_{\max} = A\omega \qquad (a_y)_{\max} = A\omega^2$$

1.5 Wave in a stretched string

Quick Quiz 1.3

The amplitude of a wave is doubled, with no other changes made to the wave. As a result of this doubling, which of the following statements is correct?

(a) The speed of the wave changes.

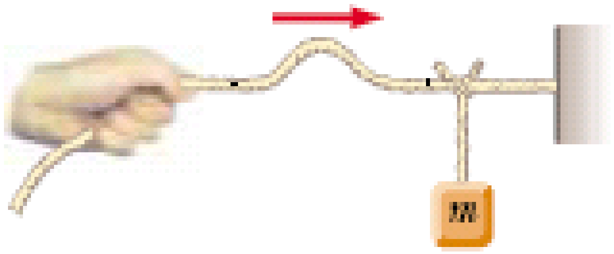
(b) The frequency of the wave changes.

(c) The maximum transverse speed of an element of the medium changes.

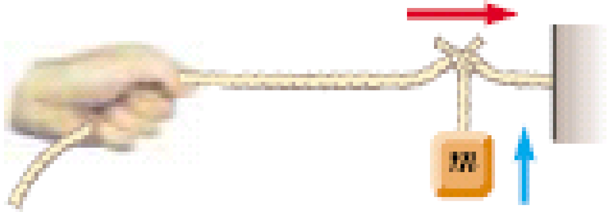
(d) All of these are true.

(e) None of these is true

1.5.3 Energy transported by a string wave:



(a)



(b)

- **Mass suspended to the string rises up when the pulse reaches it**
- **Energy is transported by a wave**
- **If we assume that no energy loss,**
- ***Total energy = constant***
- ***Kinetic energy + Potential energy = constant***

1.5 Wave in a stretched string

- **When the string at maximum displacement (y = A),**
K.E. = 0 & P.E. = maximum value

- **When the string at natural position (y = 0),**
P.E. = 0 & K.E. = maximum value = $\frac{1}{2}\ \Delta m\ (v_{max})^2$

- **T.E. = $\frac{1}{2}\ \Delta m\ (v_{max})^2 = \frac{1}{2}\ \mu\ \Delta x\ A^2\ \omega^2$**

- **Rate of energy (Power) transported, P = ΔT.E./Δt**
$P = \frac{1}{2}\ \mu\ (\Delta x/\Delta t)\ A^2\ \omega^2$

$$P = \frac{1}{2}\mu v A^2 \omega^2$$

1.5 Wave in a stretched string

Quick Quiz The amplitude of a wave is doubled, with no other changes made to the wave. As a result of this doubling, which of the following statements is correct?

(a) The speed of the wave changes.

(b) The frequency of the wave changes.

(c) The maximum transverse speed of an element of the medium changes.

(d) All of these are true.

(e) None of these is true.

Quick Quiz Which of the following, taken by itself, would be most effective in increasing the rate at which energy is transferred by a wave traveling along a string?

(a) reducing the linear mass density of the string by one half

(b) doubling the wavelength of the wave

(c) doubling the tension in the string

(d) doubling the amplitude of the wave